DOM4

$$\frac{dI}{ds} = \vec{s} \cdot \nabla I = K I_b - K I$$

$$q = \int_{4\pi} I(\vec{s})\, \vec{s}\, d\Omega = \sum w_i I_i \vec{s}_i$$

Quadrature is used in selecting the Directions $\vec{s}_i$ & weight factors w_i

- use a symmetric set invariant to rotations of 90°
- Satisfy moments $\int_{4\pi} d\Omega = 4\pi$

$$\int_{4\pi} \vec{s}\, \Omega = 0$$

$$\int_{4\pi} \vec{s}\vec{s}\, d\Omega = \frac{4\pi \delta}{3}$$

- Also, Satisfy $\int_{\vec{n}\cdot\vec{s}<0} |\vec{n}\cdot\vec{s}|\, d\Omega = \pi$ for $\vec{n} = \vec{i}, \vec{j}, \vec{k}$

See table.

$$\vec{s} = (\vec{s}_i \cdot \vec{i})\vec{i} + (\vec{s}_i \cdot \vec{j})\vec{j} + (\vec{s}_i \cdot \vec{k})\vec{k} = \xi_i \vec{i} + \eta_i \vec{j} + \mu_i \vec{k}$$

WSGG

Absorption spectrum is widely variable, full of lines.

- Very hard to get an accurate representation of radiative intensities using mean absorption coefficients.
- See slides.

WSGG

Break up / Discretize the spectrum into bands.

Assume Gray properties in each band.

$$\frac{dI_\eta}{ds} = -K_\eta I_\eta + K_\eta I_{b\eta}$$

$$\int_{\eta_j} \frac{dI_\eta}{ds} = -\int_{\eta_j} K_\eta I_\eta d\eta + \int_{\eta_j} K_\eta I_{b\eta} d\eta$$

Let $I_j = \int_{\eta_j} I_\eta d\eta$, K_η in band j (Gray) $= K_j$

$$\frac{dI_j}{ds} = -K_j I_j + K_j \underbrace{\frac{\int_{\eta_j} I_{b\eta} d\eta}{I_b}}_{a_j} * I_b$$

$$\boxed{\frac{dI_j}{ds} = -K_j I_j + K_j a_j I_b}$$

a_j is the fraction of black intensity in band j.

$$\sum a_j = 1$$

$$I = \sum I_j$$

- K_j, a_j are functions of temperature, pressure & composition.

- Bordbar's Model

  4 Gray Gases

  1 Clear Gas

  CO_2, H_2O mixtures

$$K_j = \sum_{k=0}^{4} d_{j,k} M_r^k \quad ; \quad M_r = x_{H_2O}/x_{CO_2}$$

$$a_j = \sum_{k=0}^{4} b_{j,k} T_r^k \quad ; \quad T_r = T/1200\,K$$

$$b_{jk} = \sum_{i=0}^{4} c_{j,k,i} M_r^i$$

$d_{j,k}$, $c_{j,k}$ are correlated model coefficients

  - Valid for $0.01 \le M_r \le 4$
  - for M_r outside these bounds, K_j, a_j are interpolated between pure component values for CO_2 or H_2O and the corresponding bound 0.01 or 4.

RCSLW (Rank-Correlated Spectral Line Weighted Sum of Gray Gases)

Even within a band, the gas is not gray, spectrum varies wildly.

↝ WSGGM above is more like a curve-fit for a given model formulation than an accurate formulation of the model itself given the gray assumption.

The SLW model is a WSGGM model for which the gases are gray in a given band <u>by construction</u>.

Instead of integrating over a sequential band of wavenumbers η_j, SLW integrates a set of η for which the Gas Cross Section is in a given band, which is Gray to within the width of the band

(*) $\Delta_j = \{\eta : \tilde{C}_{j-1} < C_\eta < \tilde{C}_j\}$; $\tilde{C}_j$ Denotes a Cross-Section boundary between two bands/"gases"

- Recall $K_\eta = C_\eta N$

↝ $$\frac{dI_j}{ds} = -K_j I_j + K_j \underbrace{\int_{\Delta_j} I_{b,\eta}\, d\eta}$$

evaluate this term.

- before, in WSGGM, this was written as $a_j I_b$, with $a_j = \frac{\int_{\eta_j} I_\eta d\eta}{I_b}$
- Here, it's similar; $\int_{\Delta_j} I_{b,\eta}\, d\eta = a_j I_b$, but the bounds of the integration are "over the set of η", Δ_j
- a_j is computed as

$$a_j = F(\tilde{C}_j, \phi, T) - F(\tilde{C}_{j-1}, \phi, T)$$

$$F(C, \phi, T) = \frac{1}{\sigma T^4} \int_{\{\eta : C_\eta(\eta,\phi) < C\}} E_b(\eta, T)\, d\eta$$

F is the ALBDF (absorption Line blackbody distribution function)

$\mathcal{G}$ is the fraction of black emission at some T arising from ~~Cross Sections~~ η corresponding to cross-sections below some Given Cross Section C.

- F is like a Cumulative PDF.
  - Varies from 0 to 1
  - Is monotonic in C (which allows inversion)
  - Can be computed from Spectral Databases,
- $F = F(C, \phi, T)$ where ϕ is some gas state; T, P, x_i
  - The T in ϕ can Differ from the T in $F(C, \phi, T)$, w/ the latter being the T appearing directly in $\frac{1}{\sigma T^4} \int_{\{\eta : C_\eta(\eta, \phi) < C\}} E_b(\eta, T)\, d\eta$ and the T in ϕ appearing → which Determines the $\int$ range.

In computing the K_j a logarithmic (Geometric) interpolation is used

$$K_j = N\sqrt{\tilde{C}_{j-1}\tilde{C}_j} \qquad \log K_j/N = \frac{\log \tilde{C}_{j-1} + \log \tilde{C}_j}{2} = \log\left(\tilde{C}_{j-1}\tilde{C}_j\right)^{1/2}$$

$$\rightarrow K_j = \left(C_{j-1}C_j\right)^{1/2} \cdot N$$

Spatial Effects.

The above formulation assumed spatial uniformity of composition, T, where Δ_j is independent of position.

$$\frac{dI_\eta}{ds} = -K_\eta I_\eta + K_\eta I_{b,\eta}$$

$$\underbrace{\int_{\Delta_j} \frac{dI_\eta}{ds} d\eta}_{} = -\left(\int_{\Delta_j} K_\eta I_\eta d\eta + \int_{\Delta_j} K_\eta I_{b\eta} d\eta \right)$$

$= \frac{d}{ds} \int_{\Delta_j} I_\eta \, d\eta$, that is $\frac{d}{ds}$ commutes w/ $\int$, only if the bounds of integration, $\Delta_j = \{\eta : \tilde{C}_{j-1} < C_\eta < \tilde{C}_j\}$ are independent of s, (position)

- The set of η for given $\tilde{C}$ will vary because $C(\eta)$ Dep on ~~Conf~~ ϕ, which Dep on position.
  $\Delta_j(s_1) \neq \Delta_j(s_2)$
- $\longrightarrow$ Need to account for the Leibnitz terms
  - basically "impossible"
  - big errors if you Don't.
- Soln is to reformulate so that $\Delta_j(s_1) = \Delta_j(s_2)$
  $\longrightarrow$ How? $\longrightarrow$ Δ_j is fixed $\longrightarrow$ $\tilde{C}_j$ varies w/ position.

Original method is the "Reference Approach"

Assume "ideal" behavior of the spectrum by Defining a reference state ϕ_{ref}. w/ the property:

$$\Delta = \{\eta : C_\eta(\phi_{ref}) < C^{ref}\} = \{\eta : C_\eta(\phi_{loc}) < C^{loc}\}$$

$\longrightarrow$ ALBDF at C^{loc} & C^{ref} w/ same T_b are equal.

- See figures

$$F(C^{loc}, \phi_{loc}, T_{ref}) = F(C^{ref}, \phi_{ref}, T_{ref})$$

$\longrightarrow$ invert to find C^{loc}

Method.

1. Choose ref. state ϕ_{ref}
2. Choose set $\tilde{C}_j^{ref}$ (log-spaced)
3. find local $\tilde{C}$, $\tilde{C}_j^{loc}$ by inverting $F(\tilde{C}_j^{loc}, \phi_{loc}, T_{ref}) = F(\tilde{C}_j^{ref}, \phi_{ref}, T_{ref})$
4. $K_j(s) = N_{loc} C_j^{loc} = N_{loc} \sqrt{\tilde{C}_{j-1}^{loc} \tilde{C}_j^{loc}}$
5. $a_j(s) = a_j^{loc} = F(\tilde{C}_j^{ref}, \phi_{ref}, T_b = T_{loc}) - F(\tilde{C}_{j-1}^{ref}, \phi_{ref}, T_b = T_{loc})$

ADF ref approach uses loc here.

Rank-Correlated SLW model.

2 positions s_1, s_2 w/ states ϕ_1, ϕ_2

Have Spectra $C_\eta(\eta, \phi_1), C_\eta(\eta, \phi_2)$

let $\hat{C}_1 = C_\eta(\hat{\eta}, \phi_1)$ at some Given $\hat{\eta}$

$\hat{C}_2 = C_\eta(\hat{\eta}, \phi_2)$ " " " "

Different C_j's at same η

$$H_1 = \{\eta : C_\eta(\eta, \phi_1) < \hat{C}_1\}$$
$$H_2 = \{\eta : C_\eta(\eta, \phi_2) < \hat{C}_2\}$$

; if $H_1 = H_2$, Spectra are R.C.

R.C. is less restrictive than the ideal spectrum assumption

See figure.

$$F(\hat{C}_1, \phi_1, T_r) = \frac{1}{\sigma T_r^4} \int_{H_1} E_b(\eta, T_r)\, d\eta$$

$$F(\hat{C}_2, \phi_2, T_r) = \frac{1}{\sigma T_r^4} \int_{H_2} E_b(\eta, T_r)\, d\eta$$

$H_1 = H_2 \longrightarrow F(\hat{C}_1, \phi_1, T_r) = F(\hat{C}_2, \phi_2, T_r)$

if $\hat{C}_1, \phi_1$ are given, invert to Get $\hat{C}_2, \phi_2$

Holds for arbitrary ϕ

So, if some F is given or Specified, invert to get $\tilde{C}$ at given ϕ

Method

1. Set a T_r as avg T in domain
2. Set $\tilde{F}_j$ with F_j between. These apply to all spatial locations
3. Invert: $\tilde{C}_j = C(\tilde{F}_j, \phi, T_r)$, $C_j = C(F_j, \phi, T_r)$
4. $K_j = C_j N$
5. $a_j = F(\tilde{C}_j, \phi, T_{loc}) - F(\tilde{C}_{j-1}, \phi, T_{loc})$

For mixtures

$$F(C) = F_{CO_2}(C/X_{CO_2}) F_{H_2O}(C/X_{H_2O}) F_{CO}(C/Y_{CO})$$

Examples.

- See Jupyter