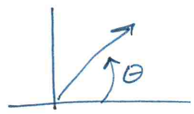
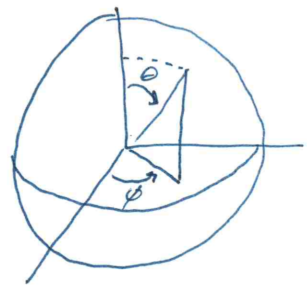
Radiation.

Intensity ; I_λ (=) energy per time per wavelength per $A_\perp$ per Solid angle.

Solid Angle ; Like the 3-D version of an angle.



Circle , 2π radians.

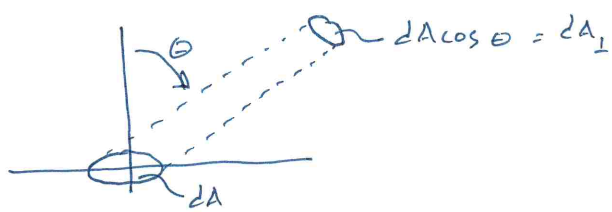


Sphere ; 4π radians.

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

$$\int_\Omega d\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} \sin\theta \, d\theta = -2\pi \left[\cos\theta\right]_0^{\pi}$$
$$= 4\pi$$

$$dA_\perp = dA\cos\theta$$



$I_\lambda \cos\theta$ is J per time per A per Solid angle

$$q_\lambda = \int_\Omega I_\lambda \cos\theta \, d\Omega$$ (=) Energy per area per wavelength through some fixed surface.

Note, $I = I(\vec{r}, \vec{s})$; location $\vec{r}$, direction $\vec{s}$

If $I \neq I(\vec{s})$

$$q_\lambda = \int_0^{2\pi}\int_0^{\pi/2} I_\lambda \cos\theta \sin\theta \, d\theta \, d\phi = 2\pi I_\lambda \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$
~ hemisphere

let $u = \cos\theta \to du = \cancel{\cos} -\sin\theta \, d\theta$

$$\to -\int u \, du = -\frac{u^2}{2} = -\frac{1}{2}\left[\cos^2\theta\right]_0^{\pi/2} = +\frac{1}{2}$$

$$q_\lambda = +\pi I_\lambda$$

* Note the Sign ; q is + in Dir of outward Surf. normal

Q: Suppose we "look" @ the Sun and it has intensity I_s
What is its intensity if it moves 10x further away from us ?

Emmission

Every medium emits radiation randomly in all directions at a rate dependent on the material properties & Temperature (Prevost's Law)

E_λ (=) energy per time per "surf" area per wavelength.

$$E = \int_\lambda E_\lambda \, d\lambda$$

Blackbody absorbs all radiation at any λ that it recieves

- At thermal equilibrium, it then must emit the same as it absorbs
- Kirchoff's Law

Planck's Law for the ~~emis~~ blackbody emissive power is given by

$$E_{b,\lambda} = \frac{C_1}{\lambda^5 \left[e^{C_2/\lambda T} - 1 \right]} \quad ; \quad C_1 = 2\pi h c_0^2 = 3.7418 \times 10^{-16} \ W \cdot m^2$$

$$C_2 = \frac{h c_0}{k} = 1.4388 \ cm \cdot K$$

- Assuming $n = 1$ (refractive index)

$$E_b = \int E_{b,\lambda} \, d\lambda = \underbrace{\left[\frac{C_1}{C_2^4} \int_0^\infty \frac{\xi^3 \, d\xi}{e^{\xi} - 1} \right]}_{\sigma = \frac{\pi^4 C_1}{15 C_2^4} = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}} T^4$$

$\sigma \equiv$ Stefan-Boltzmann Constant.

$$E_b = \sigma T^4$$

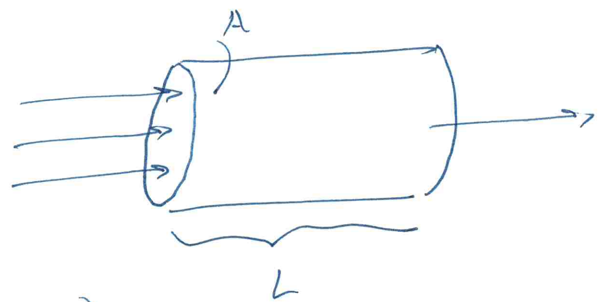
$$I_b = E_b / \pi = \sigma T^4 / \pi \quad ; \quad I_{b,\lambda} = E_{b\lambda} / \pi$$

Participating Media (PM)

- Gases, Particles
- PM absorbs, emits, Scatters.
- Gases don't really scatter at thermal λ
    - Particles like Coal, Soot aggregates do.

Absorption

Cross Section :



$f \equiv$ fraction energy blocked

$$f = \frac{A_{PM,block}}{A} = \frac{C \cdot N \cdot V}{A} = \frac{C \cdot N \cdot L \cdot A}{A} = C \cdot N \cdot L$$

$$C = \text{cross section } (=) \frac{m^2}{kmol} = \frac{m^2}{Part} \cdot \frac{Part}{kmol} = c \cdot N_a$$

$$N (=) \frac{kmol}{m^3}$$

$$L (=) m$$

$$K = f/L \ (=) \frac{1}{m} = C \cdot N$$

$K_\lambda = K_\lambda(T, P, Y_i)$; absorption coeff. is a function of T, P, Y_i

A_{PM} is the PM blockage Area in terms of the PM's interaction w/ light, not the physical area.

Radiative Transfer Equation (RTE)

here $I_\eta = I_\eta(\vec{s}, \vec{s})$
I_η is a func of location & dir Direction $\vec{s}$

$$\frac{dI_\eta}{ds} = \kappa_\eta I_{b,\eta} - \kappa_\eta I_\eta - \sigma_{s,\eta} I_\eta + \frac{\sigma_{s,\eta}}{4\pi}\int_{4\pi} I_\eta(\vec{s}_i)\, \Phi_\eta(\vec{s}_i, \vec{s})\, d\Omega$$

emission   Absorption   out-Scattering   in-Scattering

- Quasi-Steady since Speed of light is so fast Compared to other transient processes

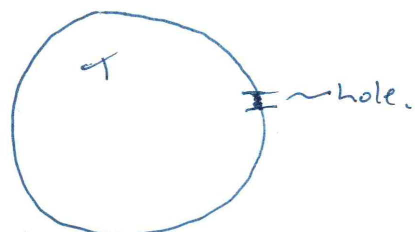
- $\frac{dI_\eta}{ds} = \vec{s} \cdot \nabla I_\eta$ (a Directional Derivative)

- η is wavenumber $= 1/\lambda$

- Note the emission term $\kappa_\eta I_{b,\eta}$

~~at rad. e Thermal Eq~~

Consider a Cavity at thermal Equilibrium. at some T

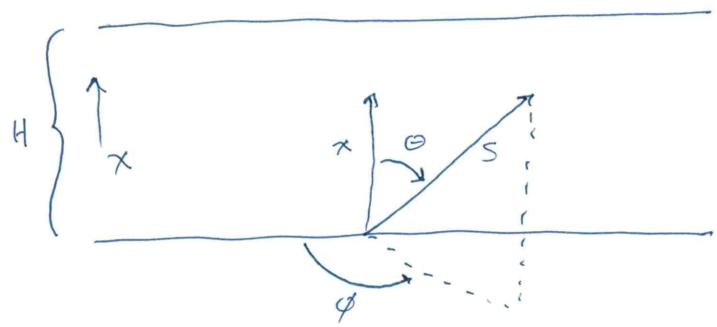


- all radiation entering is absorbed.
- then, at eq. all exiting rad is that of a black-body. (
  - any rad entering is eventually absorbed w/ a tiny probability of escaping the small hole.
  - Similarly, emitted radiation from an interior surface (not black) will absorb, reflect, etc. so that the result is a feild Dep. only on the T, and is black.
- now Consider a blob of gas placed in the center. It absorbs $\kappa_\eta I_{b,\eta}$ since $I_{b,\eta}$ is what it sees But at eq. it then has to also emit $\kappa_\eta I_{b,\eta}$

So, its emission at T is $\kappa_\eta(T) I_{b,\eta}(T)$.

Parallel Planes

- Ignore Scattering
- Use Constant K



$$\frac{dI}{ds} = KI_b - KI$$

$$I_b = \sigma T_g^4/\pi$$

$$I(s=0) = I_o = \sigma T_o^4/\pi$$

$$S\cos\theta = x$$

$$S = x/\cos\theta$$

Analytic Solution

$$I = I_b - (I_b - I_o)e^{-ks}$$

(*) $$I = I_b - (I_b - I_o)e^{-kx/\cos\theta}$$

Symmetry in ϕ

$$q = \int_{2\pi}\int_0^{\pi} I\cos\theta\sin\theta\, d\theta\, d\phi = 2\pi\int_0^{\pi} I\cos\theta\sin\theta\, d\theta$$

(*) $$q = 2\pi\int_0^{\pi/2}\cos\theta\sin\theta\left[I_b - (I_b - I_o)e^{-kx/\cos\theta}\right]d\theta -$$
$$2\pi\int_0^{\pi/2}\cos\theta\sin\theta\left[I_b - (I_b - I_o)e^{-k(H-x)/\cos\theta}\right]d\theta$$

= q Due to I w/ Positive (upward) x component. −
q Due to I w/ negative (downward) x Component

$$q_w = q(x=0) = 2\pi\int_0^{\pi/2} I_o\cos\theta\sin\theta\, d\theta - 2\pi\int_0^{\pi/2}\cos\theta\sin\theta\left[I_b - (I_b - I_o)e^{-kH/\cos\theta}\right]d\theta$$

- q_w is + if heat is leaving the wall
- q_w is − if heat is entering the wall.

$$q_w = \pi(I_o - I_b) + 2\pi(I_b - I_o)\int_0^{\pi/2} e^{-kH/\cos\theta}\cdot\cos\theta\cdot\sin\theta\, d\theta$$

is heat away from wall

(*) $$q_{w,to} = \pi(I_b - I_o) - 2\pi(I_b - I_o)\int_0^{\pi/2} e^{-kH/\cos\theta}\cdot\cos\theta\cdot\sin\theta\, d\theta$$

$$Q = -\nabla \cdot q = -\frac{dq}{dx}$$

$$Q \;(=)\; \frac{W}{m^3}$$

Q is $+$ for local radiative addition $\longrightarrow$ increase T

$\nabla \cdot q$ is net "flux" out per unit volume.

$$Q = -2\pi \int_0^{\pi/2} k \sin\theta \,(I_b - I_0)\, e^{-kx/\cos\theta}\, d\theta$$
$$\quad -2\pi \int_0^{\pi/2} k \sin\theta \,(I_b - I_0)\, e^{-k(H-x)/\cos\theta}\, d\theta$$

(*) $$Q = -2\pi k (I_b - I_0) \int_0^{\pi/2} \sin\theta \left(e^{-kx/\cos\theta} + e^{-k(H-x)/\cos\theta} \right) d\theta.$$

Emissivity, mean beam length.

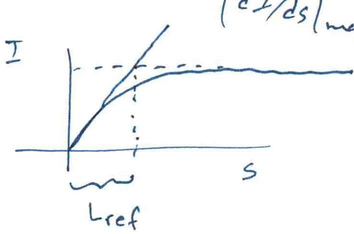
$$\frac{dI}{d\xi} = -kI + kI_b \longrightarrow I = I_b - (I_b - I_0)\cdot e^{-kL'}$$

Let $I_0 = 0$

$$I = I_b - I_b e^{-kL} = I_b \left(1 - e^{-kL}\right)$$

$$\epsilon = \frac{I}{I_b} = 1 - e^{-kL}$$

- Isothermal, constant k
- $1/k$ is the characteristic length scale.
- $$L_{ref} = \frac{\Delta I}{|dI/ds|_{max}} = \frac{\cancel{I_b - I_0}}{k\,\cancel{(I_b - I_0)}} = \frac{1}{k}$$



- for given $\epsilon = \frac{I}{I_b}$, & given k, L is a mean beam length.
- $1/k$ Determines the optical thickness.

Planck Mean Absorption Coefficient.

$$K_{PM} = \frac{\pi}{\sigma T^4}\int_\eta I_{b,\eta} K_\eta \, d\eta = \frac{\int_\eta I_{b,\eta} K_\eta \, d\eta}{\int I_{b,\eta} \, d\eta} = \frac{\int_\eta I_{b,\eta} K_\eta \, d\eta}{\sigma T^4 / \pi}$$

$K_{PM} = K_{PM}(T)$ for given Species.

$$K_{PM,i} = x_i P a_i(T)$$

x_i (=) mole fraction
P (=) Pressure
$a_i(T)$ (=) Polynomial fit.

$$K_{mix} = \sum K_{PM,i}$$

See Plots.

Optically - thin approximation.

$$\frac{dI}{ds} = kI_b - kI$$

I_b is the black intensity of some gas; $I_b = \sigma T_g^4/\pi$

I has some initial conditions I_o from the surroundings

Take $I_o = \sigma T_\infty^4 / \pi$

Now, $\frac{dI}{ds} = \vec{s} \cdot \nabla I$

$$\int_\Omega \vec{s} \cdot \nabla I \, d\Omega = \nabla \cdot \int_\Omega I \vec{s} \, d\Omega = \nabla \cdot \vec{q}$$

$$\rightsquigarrow \nabla \cdot \vec{q} = \int_\Omega k I_b \, d\Omega - \int_\Omega k I \, d\Omega$$

- k is indep of Direction
- Assume Isotropic I_b, I

$$\nabla \cdot \vec{q} = 4\pi k (I_b - I)$$

Now, evaluate this for $s \ll 1/k \rightarrow$ optically thin

so $I = I_o$

$$\nabla \cdot \vec{q} = 4\pi k (I_b - I_o)$$

$$\nabla \cdot \vec{q} = 4k\sigma (T_g^4 - T_\infty^4)$$

$$Q = -\nabla \cdot \vec{q}$$

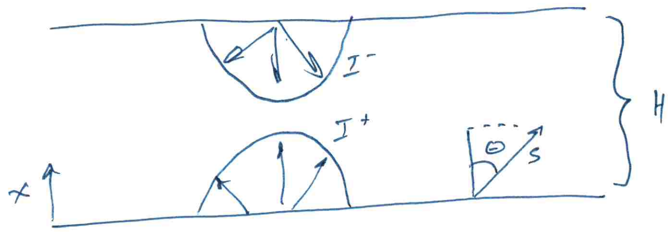
$$\boxed{Q = -4k\sigma (T_g^4 - T_\infty^4)}$$

is radiative heat loss.

$Q > 0 \rightsquigarrow$ energy is lost, T_g Decreases

2 Flux Model.

$$\frac{dI}{ds} = kI_b - kI$$



$x = s\cos\theta$

$s = x/\cos\theta$

(1) $$\frac{dI\cos\theta}{dx} = kI_b - kI$$

Assume I is indep. of Direction in each hemisphere.

I^+, I^-

we have eq (1) for each θ

The average intensity then corresponds to the avg $\cos\theta$ over the hemisphere.

$$\overline{\cos\theta} = \frac{\int \cos\theta\, d\Omega}{\int d\Omega} = \frac{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta\, d\theta\, d\phi}{2\pi}$$

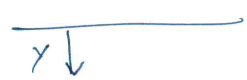
$$= \frac{1}{2}$$

$$\frac{dI^+}{dx} = 2kI_b - 2kI^+$$

$$I^+(0) = I_o$$

Solution: $I^+ = I_b + (I_o - I_b)e^{-2kx}$

Now, I^- follows the same eqns but from the top Plate



$$I^- = I_b + (I_o - I_b)e^{-2ky}$$

but $y = H - x$

$$I^- = I_b + (I_o - I_b)e^{-2k(H-x)},$$

Finally, both I^+ and I^-, as above, are positive, but I^+ is Directed upward and I^- is Directed Downward, so to be Consistent, Replace I^- with $-I^-$

$$I^- = -I_b - (I_o - I_b)e^{-2k(H-x)}$$

Again: $I^+ = I_b + (I_0 - I_b)e^{-2kx}$

$I^- = -I_b - (I_0 - I_b)e^{-2k(H-x)}$

$$I = I^+ + I^- = (I_0 - I_b)\left(e^{-2kx} - e^{-2k(H-x)}\right)$$

$$* \quad q = \pi I = \pi(I_0 - I_b)\left(e^{-2kx} - e^{-2k(H-x)}\right)$$

$$* \quad Q = -\frac{dq}{dx} = 2\pi k(I_0 - I_b)\left(e^{-2kx} + e^{-2k(H-x)}\right)$$

$$* \quad q_w = \pi(I_0 - I_b)\left(1 - e^{-2kH}\right)$$

is q away from the wall.

$*$ -1 to get q to the wall.