PPDF

Equilibrium

$\phi = \phi(\xi)$ via equilibrium.

$$\tilde{\phi} = \int \phi(\xi)\, \tilde{P}(\xi)\, d\xi$$

$$\tilde{P}(\xi) = \frac{\rho(\xi)\, P(\xi)}{\bar{\rho}}$$

$$\bar{\rho} = \int \rho(\xi)\, P(\xi)\, d\xi$$

$P(\xi) = P(\xi; \tilde{\xi}, \sigma^2)$ = β-PDF (or clipped Gaussian.)

$$P_\beta(\xi; \tilde{\xi}, \sigma^2) = \xi^{a-1}(1-\xi)^{b-1} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$$

$$a = \frac{\tilde{\xi}(\tilde{\xi} - \tilde{\xi}^2 - \sigma^2)}{\sigma^2}$$

$$b = \frac{1-\tilde{\xi}}{\tilde{\xi}} \cdot a$$

$$\sigma^2 \equiv \widetilde{\xi''^2}$$

$$\tilde{\phi} = \tilde{\phi}(\tilde{\xi}, \sigma^2) = \int \phi(\xi)\, \tilde{P}_\beta(\xi; \tilde{\xi}, \sigma^2)\, d\xi$$

CFD: Transport $\tilde{\xi}$

Transport or model σ^2

Lookup $\tilde{\phi}(\tilde{\xi}, \sigma^2)$ from Pre-tabulation

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PPDF

Equilibrium with Heat loss

- use $\gamma = \dfrac{h_a(\xi) - h}{h_s(\xi)}$

  - $h_a(\xi) = h_{\xi=0} \cdot (1-\xi) + h_{\xi=1} \cdot (\xi)$
  - $h_s(\xi) = h_a(\xi) - h(Y_a(\xi), T_{mix}(\xi))$

    $T_{mix}(\xi) = T_{mix}(Y_{mix}(\xi), h_a(\xi))$

    $Y_{mix}(\xi) = Y_{\xi=0}(1-\xi) + Y_{\xi=1} \cdot (\xi)$

- $\tilde{\phi} = \tilde{\phi}(\tilde{\xi}, \sigma^2, \gamma)$
- $\tilde{\phi} = \iint \phi(\xi,\gamma)\tilde{P}(\xi,\gamma)\,d\xi\,d\gamma$

  $\approx \int \phi(\xi,\gamma)\tilde{P}(\xi)\,d\xi$

  $\tilde{P}$ treated as before.

Neglecting fluctuations in γ

- Implies a subgrid structure in which a given γ applies to all subgrid ξ's

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Tabulate $\tilde{\phi}(\tilde{\xi}, \sigma^2, \gamma)$

- ~~Equilibrium with heat loss,~~

$\tilde{\phi}(\tilde{\xi}, \sigma^2, \gamma) = \int \phi(\xi,\gamma)\tilde{P}(\xi; \tilde{\xi}, \sigma^2)\,d\xi$

$\phi(\xi,\gamma)$ from equilibrium with heat loss.

CFD $\rightarrow \tilde{\xi}, \sigma^2, \tilde{h}$

$\tilde{\xi} \rightarrow h_a, h_s$

$\tilde{h}, h_a, h_s \rightarrow \gamma = \dfrac{h_a - \tilde{h}}{h_s}$

Lookup $\tilde{\phi}(\tilde{\xi}, \sigma^2, \gamma)$

$\tilde{\xi}$ equation

$$\frac{\partial \bar{\rho}\tilde{\xi}}{\partial t} + \nabla\cdot(\bar{\rho}\tilde{v}\tilde{\xi}) = \cancel{\underbrace{\nabla\cdot(\bar{\rho} D \nabla\tilde{\xi})}_{\text{molecular transport}}} - \underbrace{\nabla\cdot(\bar{\rho}\widetilde{v''\xi''})}_{\text{turbulent transport}}$$

Closure

- $\widetilde{v''\xi''} = -D_t \nabla \tilde{\xi}''$

$$D_t = \frac{\nu_t}{Sc_t}$$

- Normally Neglect molecular transport, or neglect the term and replace $D_t \to D + D_t$ in the turbulent transport model.

$\widetilde{\xi''^2}$ equation

$$\frac{\partial \bar{\rho}\widetilde{\xi''^2}}{\partial t} + \nabla\cdot(\bar{\rho}\tilde{v}\widetilde{\xi''^2}) = -\nabla\cdot(\bar{\rho}\widetilde{v''\xi''^2}) - 2\bar{\rho}(\widetilde{v''\xi''})\cdot\nabla\tilde{\xi} - \bar{\rho}\,\underbrace{\widetilde{2D\nabla\xi''\cdot\nabla\xi''}}_{\tilde{\chi}}$$

(A) turbulent transport
(B) Production
(C) Dissipation

- neglects molecular Diffusion

Closure

(A) $\widetilde{v''\xi''^2} = -D_t \nabla \widetilde{\xi''^2}$

(B) $\widetilde{v''\xi''}\cdot\nabla\tilde{\xi} = -D_t(\nabla\tilde{\xi}\cdot\nabla\tilde{\xi})$

(C) $\tilde{\chi} = C_\chi \dfrac{\tilde{\epsilon}}{\tilde{k}}\cdot\widetilde{\xi''^2}$, $C_\chi = 2$

See Peters p. 29, 30, 31, 32, 194

See Poinsot p 317 - 319

LES.

Model $\widetilde{\xi''^2} = C_\xi \cdot \Delta^2(\nabla\tilde{\xi}\cdot\nabla\tilde{\xi})$ See ~~Pei~~ Pierce 1998 PoF

C_ξ evaluated using the Dynamic Smagorinsky model

Model $\widetilde{\xi''} = C\cdot(\widetilde{\xi^2} - \tilde{\tilde{\xi}}^2)$ w/ $C=1$ $-$, $\sim$ Refer to filter widths.   See Cook & Riley 1994