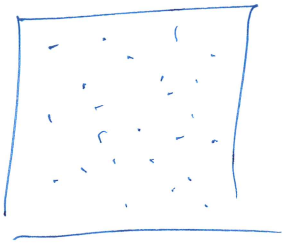
Mixing Models.



Specialize here to transported Scalars (as opposed to velocities)

Timescale (RAS)

$$\tau_\phi \equiv \frac{\widetilde{\phi''^2}}{2\epsilon_\phi} \quad \text{(Scalar mixing timescale)}$$

$$\tau_\phi = \frac{\tau}{C_\phi} = (C_\phi \omega)^{-1} ; \qquad \tau = k/\epsilon \; ; \; \omega = \epsilon/k = 1/\tau$$

τ is the turbulence timescale

$C_\phi = 2.0$ (usually) (1-3 reported)

Wanted Properties of MM

- Pope chp 12
- Fox Chap 6
- 122 C&F 1998 115:487-514 EMST.

(1) mixing preserves the mean

(2) Scalar variance Decay at correct rate

(3) Scalars remain Bounded

Others: . Pope 1983 26:404-408 Phys Fluids (linearity / indep. props)

- Homogeneous → PDF relax to joint Normal Juneja & Pop 1996 ; 8:2161 Phys Fluids
- Respect Locality in Composition Space.

Most models Do Not address Diff Diff

Do Not Account for turbulence length-scale influences

Do Not include a Direct influence of Chemical Rxn on mixing

2 categories.

① Particles are indep. of each other. (weakly interact through the mean)

② Direct interactions.

IEM : Given Particle i

$$\frac{d\phi_i}{dt} = -\frac{1}{2}\frac{(\phi_i - \bar{\phi})}{\tau_\phi} \longrightarrow \phi_i(t) = \bar{\phi}_i + (\phi_i(0) - \bar{\phi})e^{-t/2\tau_\phi}$$

MC

- Select particle pairs randomly at Rate $1/\tau_\phi$ $\to N\frac{\Delta t}{\tau_\phi}$ | (p, q) pair
- Change Compositions to:

$$\phi_{p,new} = \phi_p + \frac{1}{2}r(\phi_q - \phi_p)$$
$$\phi_{q,new} = \phi_q + \frac{1}{2}r(\phi_p - \phi_q)$$

$r \in (0,1)$ is random number.

Suppose $r=1$, $\phi_p = 1$, $\phi_q = 0$

$$\to \phi_{p,new} = 1 + \frac{1}{2}(1)(0-1) = \frac{1}{2}$$
$$\phi_{q,new} = 0 + \frac{1}{2}(1)(1-0) = 1/2$$

EMST.

- Complex
- Account for "locality" in Composition Space

2-Particle mixing $\quad A \rightleftarrows B$

$$\frac{dA}{dt} = \frac{B-A}{\tau}$$

$$\frac{dB}{dt} = \frac{A-B}{\tau} = -\frac{dA}{dt}$$

Let $z = \frac{dA}{dt} \longrightarrow \frac{dz}{dt} = \frac{1}{\tau}\frac{dB}{dt} - \frac{1}{\tau}\frac{dA}{dt} = -\frac{2}{\tau}\frac{dA}{dt} = -\frac{2}{\tau} z$

$\frac{dz}{z} = -\frac{2}{\tau}\,dt$ (separate variables)

$\ln z = -\frac{2}{\tau} t + c$ (integrate)

$\rightarrow z = z_0 e^{-2t/\tau}$ (rearrange, apply initial cond. to eval c)

$\rightarrow \frac{dA}{dt} = A_0' e^{-2t/\tau}$

$A = A_0' \int e^{-2t/\tau} dt = A_0' \left[-\frac{\tau}{2} e^{-2t/\tau} + c \right]$

@ $t=0$ $\quad A = A_0 \longrightarrow c = \frac{A_0}{A_0'} + \frac{\tau}{2}$

$A = A_0 + \frac{\tau}{2} A_0' \left(1 - e^{-2t/\tau}\right)$

$A_0' = \frac{B_0 - A_0}{\tau}$

$$\boxed{\begin{aligned} A &= A_0 + \frac{B_0 - A_0}{2}\left(1 - e^{-2t/\tau}\right) \\ B &= B_0 + \frac{A_0 - B_0}{2}\left(1 - e^{-2t/\tau}\right) \end{aligned}}$$

- @ $t=0$ $\quad A_0 = A, \quad B_0 = B$
- @ $t = \infty$ $\quad \left.\begin{aligned} A &= \frac{A_0 + B_0}{2} \\ B &= \frac{B_0 + A_0}{2} \end{aligned}\right\} A = 0$

$\rightarrow$ Good