in Fig. 1.7(b) is

$$A_{rb} = \frac{1}{h^2}\begin{pmatrix} -1 & 0 & -1 & & & & & \\ -1 & -1 & 0 & -1 & & & & \\ -1 & 0 & -1 & -1 & -1 & & & \\ & -1 & 0 & -1 & 0 & -1 & & \\ & & -1 & 0 & -1 & 0 & -1 & \\ & & & -1 & -1 & -1 & 0 & -1 \\ & & & & -1 & 0 & -1 & -1 \\ & & & & & -1 & 0 & -1 \end{pmatrix}. \tag{1.4.5}$$

Remark 1.4.1 As we will see below (Section 1.5), in a multigrid algorithm it is usually not necessary to build up the matrix A_h coming from the discretization. The multigrid components are based on "local" operations; multiplications and additions are carried out grid point by grid point. The storage that is needed in a multigrid code mainly consists of solution vectors, defects and right-hand sides on all grid levels. $\gg$

1.4.2 Poisson Solvers

Table 1.1 gives an overview on the complexity of different solution methods (including fast Poisson solvers) applied to Model Problem 1. Here direct and iterative solvers are listed. For the iterative solvers, we assume an accuracy (stopping criterion) in the range of the discretization accuracy. This is reflected by the $\log \varepsilon$ term. The full multigrid (FMG) variant of multigrid which we will introduce in Section 2.6 is a solver up to discretization accuracy.

It is generally expected that the more general a solution method is, the less efficient it is and vice versa. Multigrid is, however, a counter example for this pattern—indeed multigrid

Table 1.1. Complexity of different solvers for the 2D Poisson problem (N denotes the total number of unknowns).

Method	# operations in 2D
Gaussian elimination (band version)	$O(N^2)$
Jacobi iteration	$O(N^2 \log \varepsilon)$
Gauss–Seidel iteration	$O(N^2 \log \varepsilon)$
Successive overrelaxation (SOR) [431]	$O(N^{3/2} \log \varepsilon)$
Conjugate gradient (CG) [194]	$O(N^{3/2} \log \varepsilon)$
Nested dissection (see, for example, [9])	$O(N^{3/2})$
ICCG [264]	$O(N^{5/4} \log \varepsilon)$
ADI (see, for example, [403])	$O(N \log N \log \varepsilon)$
Fast Fourier transform (FFT) [112]	$O(N \log N)$
Buneman [93]	$O(N \log N)$
Total reduction [342]	$O(N)$
Multigrid (iterative)	$O(N \log \varepsilon)$
Multigrid (FMG)	$O(N)$