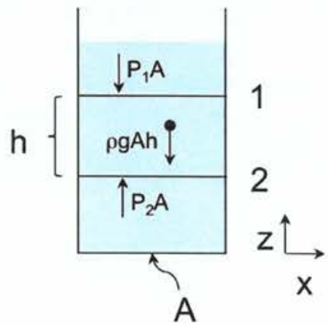
ChE 374–Lecture 3–Fluid Statics

- Review of pressure
  - P = F/A (P is a stress)
    - Scalar: same in all directions
    - Acts normal to surfaces
  - Units: SI: Pa = kg/ms^2, Eng: lbf/in^2 or psi
    - 1 atm = 101325 Pa = 14.7 lbf/in^2 (MEMORIZE THIS)
  - Absolute versus Gage pressure
    - Gage is relative to the atmosphere. Very practical
    - Absolute must always be used in the I.G. law.
    - Always specify which!
- Barometric equation
  - Describes pressure variation in a fluid



  - FORCE BALANCE
  - $\Delta P = \rho g h$
    - (for constant ρ)
  - $\frac{dP}{dz} = -\rho g.$
    - Be careful of signs (z points up), P increases with depth.
    - No horizontal changes in pressure, because there is no horizontal gravity..
      - P in same fluid at same height is the same!
      - All that matters is how much fluid is above.
    - More generally: $\frac{dP}{dz} = -\rho g \rightarrow \nabla P = \rho \vec{a}.$
- Example: Pressure at the bottom of a pool
  - Water vs. Air
    - Ignore variation of P with h for air for short distances.
      - Not true for air flow with density variations (like chimneys).
      - True for liquids exposed to the atmosphere → same P even if at different heights.
- Example: Variation of P in the atmosphere
  - Density varies.
    - Use ideal gas law.
    - How does temperature vary → three cases: (1) isothermal, (2) adiabatic, (3) average of measurements.

Class 3 - Fluid Statics

- $P = \frac{F}{A}$ F is the "normal" Force per area.

- P is isotropic : Same in all Directions.

Same pressure on plate in 3 configs.

- Pressure from molecular collisions, which are random and in all Directions.

Units

$\frac{F}{A} = \frac{N}{m^2} = Pa = \frac{kg}{ms^2}$ or $\frac{lbf}{in^2} = psi$

1 atm = 101325 Pa = 14.7 psi

Absolute vs Gage

Abs → P relative to vacuum

Gage → P relative to atmosphere

$P_g = P_{abs} - P_{atm}$

$P_{abs} = P_g + P_{atm}$

- Always Specify
  psi vs psia vs psig

- Thermodynamics uses P_{abs} : Ideal Gas $P = \rho RT/M$ (P is P_{abs})

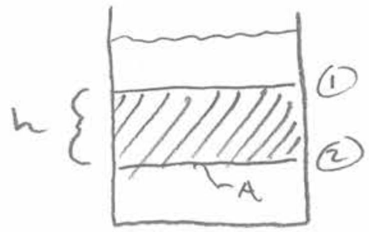
Barometric Eqn

- Pressure Statics → No Fluid Motion

- $\boxed{\frac{dP}{dz} = -\rho g}$ $\boxed{\Delta P = \rho g h}$ Know These.

Details →

Force Balance: $\sum F = 0$

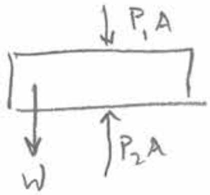


X-Dir: Pressure on Sides → Cancels.

Z-Dir: Forces

* Q: How many Forces? → 1, 2, 3, 4?

→ 3: P_1, P_2, weight.



$W = \rho A h g$

$\sum F = 0 \longrightarrow P_2 A - P_1 A - \rho A h g = 0$

$P_2 - P_1 = \rho g h \longrightarrow \boxed{\Delta P = \rho g h}$

- Pressure increases with Depth h
- h measured Down
- Assumed Const ρ
  - If ρ Not Const:

Shrink to a point: $h = z_1 - z_2 = -\Delta z$

$= -(z_2 - z_1)$

$\Delta P = \rho g h = -\rho g \Delta z \longrightarrow \dfrac{\Delta P}{\Delta z} = -\rho g$

$\lim \Delta z \to 0 \longrightarrow \boxed{\dfrac{dP}{dz} = -\rho g}$ $\left(\text{or } \vec{\nabla} P = \rho \vec{a}\right)$

- Z points up!

- No Horizontal ΔP cause No horizontal gravity
  - P in Same fluid at Same height is Same.

Example 1

Force at Bottom of Pool

$$\Delta P = \rho g h = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(3\ m) \approx 30{,}000\ Pa$$

$$\frac{\Delta P}{P_{atm}} = 30\%$$

Air instead? $\rightarrow$ $\frac{\rho_{air}}{\rho_{water}} = \frac{1.2}{1000}$ $\rightarrow$ $\frac{\Delta P}{P_{atm}} \approx 0.03\%$

$\rightarrow$ Ignore variation of P with h for air for Short Distances

- when Considering liquids at Different heights, ignore the weight of the air.
- Not True for Hot gas vs Cool Gas
  - Chimney
  - Hot air balloon
  - Gas 1 vs Gas 2

  } $\Delta \rho$ matters here.

Example 2

Atmosphere: ρ Changes with height

* Quiz See Slide

- e.g. Get an expression for $P(z)$ in atmosphere
- Assume Const g $\rightarrow$ why? (Earth D ≈ 8000 miles, Atmosphere ≈ 20 miles Thick)
- Assume Isothermal

$$\frac{dP}{dz} = -\rho g \quad ; \quad \rho = MP/RT$$

$$\frac{dP}{dz} = -\alpha P \quad ; \quad \alpha = Mg/RT \text{ is Const.}$$

Integrate with B.C. $P = P_{atm}$ at $z = 0$

$$\frac{dP}{P} = -\alpha\, dz \qquad \text{(sep. vars)}$$

$$\int_{P_{atm}}^{P} \frac{dP}{P} = \int_0^z -\alpha\, dz' \longrightarrow \ln\frac{P}{P_{atm}} = -\alpha z$$

$$\longrightarrow P = P_{atm} \exp(-\alpha z)$$

$$* \quad P = P_{atm} \exp(-Mgz/RT)$$

Optional:

__Isentropic:__ As gas expands, it does work on surrounding gas, but loses no heat
→ Temperature drops

Next Semester: $T = T_1 \cdot \left(\frac{P}{P_1}\right)^{\alpha}$; $\alpha = \frac{\gamma - 1}{\gamma}$, $\gamma = \frac{C_p}{C_v}$

$\gamma = 1.4$ for air (assumed const)

$$\frac{dP}{dz} = -\frac{MPg\, P_1^{\alpha}}{RT_1\, P^{\alpha}} = \underbrace{-\frac{MgP_1^{\alpha}}{RT_1}}_{c} \cdot P^{1-\alpha}$$

$$\frac{dP}{dz} = -cP^{1-\alpha} \longrightarrow \frac{dP}{P^{1-\alpha}} = -c\,dz \qquad = P^{\alpha-1}dP$$

$$\int_{P_{atm}}^{P} P^{\alpha-1} dP = -\int_0^z c\, dz$$

Solve, insert c:

$$* \quad P_2 = P_{atm}\left[1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{Mg}{RT_{atm}}(z_2 - z_1)\right]^{\gamma/\gamma-1}$$

$$\longrightarrow T_2 = T_{Atm}\left(1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{Mg}{RT_{atm}}(z_2)\right)$$

form is $T = T^* + Bz$ is a fit

The "Standard" Atmosphere

$$T = T^* + Bz \qquad T^* = 288.15$$
$$B = -0.0064876 \text{ K/m}$$

$$\frac{dP}{dz} = -\frac{MPg}{R(T^* + Bz)}$$

$$\frac{dP}{P} = -\frac{Mg}{R(T^* + Bz)} dz$$

* Solve → $P_2 = P_1 \left[\frac{T^* + Bz}{T^*} \right]^{-Mg/BR}$

See Slide: Plots P_{iso}, P_{isen}, P_{std} vs z

Also T_{iso}, T_{isen}, T_{std}.

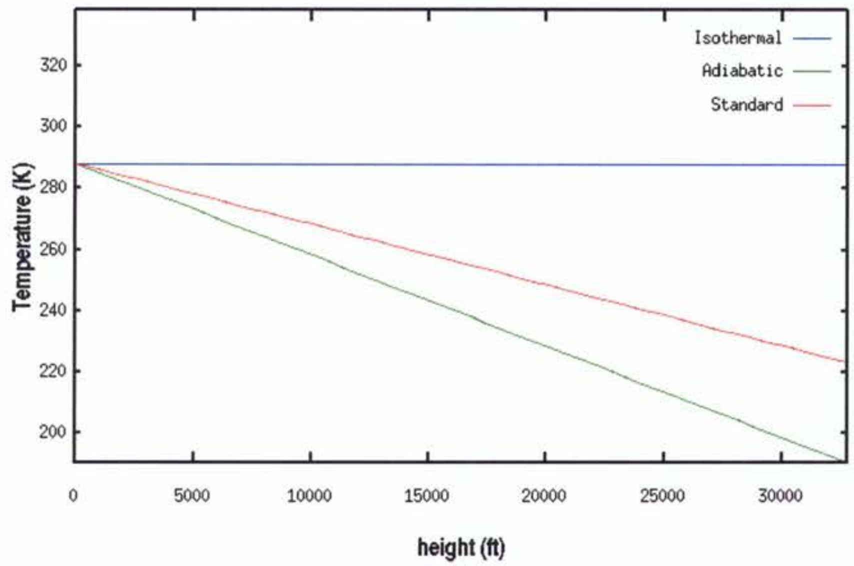
- P at 30,000 ft = 0.34 atm → 34% !
  = Everest

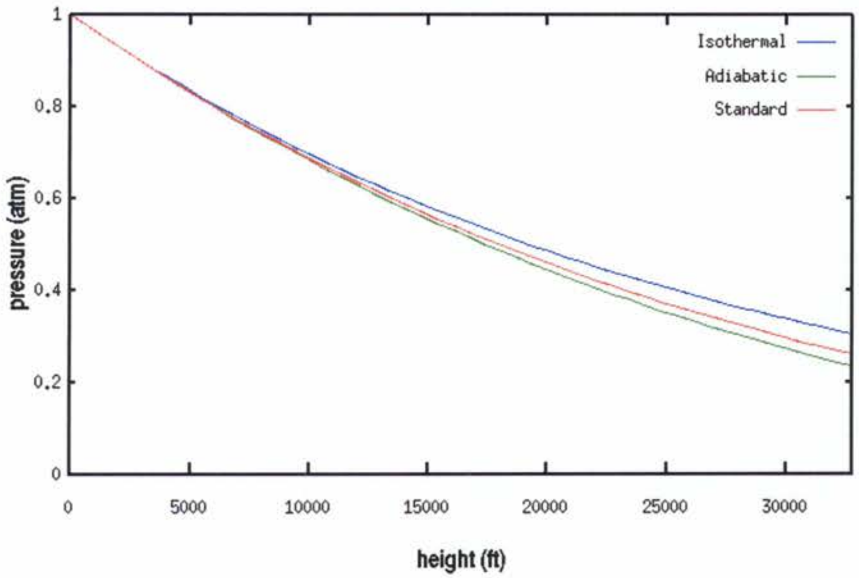
$T_{std} \sim 240$ K = −28 °F

Quiz

- Develop and expression for the pressure in the atmosphere as a function of elevation:
  - P(z)
  - Where $P(0) = P_{atm}$.
  - Assume constant temperature T.
- Strategy
  - Go with what you know:
  - What do you expect to happen?
  - Draw a picture
  - What is an expression or law that relates the physics?
  - What additional information or assumptions are needed.
  - Talk to your neighbor ☺

Temperature, Pressure in Atmosphere





Ht (ft)	P iso (atm)	P adia	P std
0.00e+00	1.000e+00	1.0000e+00	1.0000e+00
1.7268e+03	9.3941e-01	9.3888e-01	9.3906e-01
3.4537e+03	8.8250e-01	8.8048e-01	8.8117e-01
5.1805e+03	8.2903e-01	8.2472e-01	8.2620e-01
6.9074e+03	7.7880e-01	7.7153e-01	7.7405e-01
8.6342e+03	7.3161e-01	7.2084e-01	7.2459e-01
1.0361e+04	6.8729e-01	6.7257e-01	6.7773e-01
1.2088e+04	6.4565e-01	6.2665e-01	6.3336e-01
1.3815e+04	6.0653e-01	5.8302e-01	5.9137e-01
1.5542e+04	5.6978e-01	5.4160e-01	5.5166e-01
1.7268e+04	5.3526e-01	5.0233e-01	5.1414e-01
1.8995e+04	5.0283e-01	4.6514e-01	4.7872e-01
2.0722e+04	4.7236e-01	4.2995e-01	4.4530e-01
2.2449e+04	4.4374e-01	3.9671e-01	4.1380e-01
2.4176e+04	4.1686e-01	3.6535e-01	3.8413e-01
2.5903e+04	3.9160e-01	3.3580e-01	3.5621e-01
2.7629e+04	3.6788e-01	3.0800e-01	3.2995e-01
2.9356e+04	3.4559e-01	2.8188e-01	3.0529e-01
3.1083e+04	3.2465e-01	2.5738e-01	2.8214e-01
3.2810e+04	3.0498e-01	2.3444e-01	2.6044e-01