Turbulent Scaling.

Concepts: Large Scales, Small Scales.
Cascade
Isotropy
Energy Spectrum

- Integral (Large) Scales: l_o, u_o, τ_o
- Kolmogorov Scales (small): η, u_η, τ_η
- Turbulence Deals with fluctuations: $V = \bar{V} + V'$
  but V' is (nominally) Zero in the mean: $\overline{V'} = 0$
  → Consider V'^2
  → Turbulent Kinetic Energy is $\frac{1}{2} V'^2$ (=) $\frac{m^2}{s^2}$
- Energy Dissipation Rate: ϵ (=) $\frac{m^2}{s^3} = \left(\frac{m^2}{s^2}\right)/s$
- Kinematic Viscosity $\nu = \frac{m^2}{s} = \mu/\rho$
- Kolmogorov Hypothesis: See Pope (2000) Turbulent Flows.

① • Kolmogorov's hypothesis of local Isotropy:

At sufficiently high Re, the small scale turbulent motions: $l \ll l_o$ are Statistically isotropic

② • Kolmogorov's first similarity hypothesis:

In every turbulent flow at suffiently high Re, the Statistics of the Small Scale motions have a universal form that is uniquely determined by ν and ϵ

③ • Kolmogorov's Second Similarity hypothesis:

In every turbulent flow at Sufficiently high Re, the Statistics of the motions of Scale l in the range $l_o \gg l \gg \eta$ have a universal form That is uniquely determined by ϵ, independent of ν

① → Directional info is lost in the cascade process

→ All info about the Geometry of the large eddies - Determined from the mean flow field & Boundary Conditions is lost.

→ Small Scale motions are <u>universal</u> : Similar in every high Re turbulent flow.

② → $\epsilon, \nu \rightsquigarrow \eta, u_\eta, \tau_\eta$

$\epsilon \,[=]\, \frac{m^2}{s^3}$, $\nu \,[=]\, \frac{m^2}{s}$

$\eta \,[=]\, m \rightarrow \frac{\nu^3}{\epsilon}$ Cancels $s \,[=]\, m^4 \rightarrow \left(\frac{\nu^3}{\epsilon}\right)^{1/4} [=]\, m$

- $\eta \equiv \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$
- $u_\eta \equiv (\epsilon\nu)^{1/4}$ $\qquad Re_\eta = \frac{\eta u_\eta}{\nu} = 1$
- $\tau_\eta \equiv (\nu/\epsilon)^{1/2}$ $\qquad \epsilon = \nu\left(\frac{u_\eta}{\eta}\right)^2 = \frac{\nu}{\tau_\eta^2}$

$\frac{u_\eta}{\eta} = \frac{1}{\tau_\eta}$ ~ Velocity Gradients of Dissipative Scales.

- Let $Re = \frac{l_0 u_0}{\nu}$, $\epsilon \sim u_0^3/l_0 = u_0^2/\tau_0 = u_0^2/(l_0/u_0)$

→
- $\frac{\eta}{l_0} \sim Re^{-3/4}$
- $\frac{u_\eta}{u_0} \sim Re^{-1/4}$
- $\frac{\tau_\eta}{\tau_0} \sim Re^{-1/2}$

→ $\tau_\eta << \tau_0$ at high Re

→ Small Scales have small timescales

→ Adjust "quickly" to their environment.

→ ~ Equilibrium

→ $\eta << l_0$ at high Re

→ Range of Scales

→ An intermediate Range $l >> \eta$ so $l << l_0$

→ These intermediate scales will have large Re

→ indep. of Viscosity → (3)

→ The range of τ implies that large Scales (high τ) Are Dynamically Decoupled from Small Scales (low τ)

- Scale interactions are dynamically local,

  → large Scales Don't immediately break down to small Scales, rather, there is a Cascade of Scales

  $1 \to \frac{1}{2} \to \frac{1}{4} \to \frac{1}{8}$ or something: a nominally multiplicative Decrease.

(3) → At intermediate Scales, ϵ ~ constant

- Energy in = Energy out : These are The inertial Range Scales. These are bigger than η, so not Dissipative, but Smaller than l_0 → universal, isotropic.
- LES Resolves into this range.

$\rightarrow$ Scales Depend on ϵ alone

$$\epsilon = \frac{m^2}{s^3} \rightarrow \epsilon \sim \frac{l^2}{\tau^3}$$

$\rightarrow$
- $\tau \sim l^{2/3}$
- $l \sim \tau^{3/2}$
- $u \sim l^{1/3}$

- Again, Smaller Scales have Smaller τ = higher rate
- Smaller Scales have Smaller velocity $\rightarrow$ Small Scales are Swept by The large Scales.

Consider Diffusion: if we call $\frac{d\langle x^2\rangle}{dt} \sim D$ a turbulent Diffusivity, where $\langle x^2\rangle$ is The mean fluid parcel Displacement, Then $D (=) \frac{m^2}{s} = \frac{l^2}{\tau} \sim \frac{l^2}{l^{2/3}} = l^{4/3}$

So, larger Scales have a higher "turbulent Diffusivity" than Small Scales.

Note, the above Scaling is The basis for turbulent models, ODT, LEM, HiPS, turb. Diffusion based models.

$$E \sim \frac{u^2}{K} \rightarrow u^2 \cdot l$$

$$\left.\begin{array}{l} \epsilon \sim \frac{u^2}{\tau} \rightarrow \cancel{E} \; E \rightarrow \tau \cdot l \\ \epsilon \sim \text{const} \\ \epsilon \sim \frac{l^2}{\tau^3} \rightarrow \tau \sim l^{2/3} \end{array}\right\} \rightarrow E \sim l^{2/3} \cdot l \sim l^{5/3} \sim \underline{\underline{K^{-5/3}}}$$