Discretization, Errors

$$\left(\frac{\delta\phi}{\delta x}\right)_j = \frac{\sum_k a_k \phi_{j+k}}{\Delta x}$$

Position, k	a_k			
	2^{nd}	4^{th}	6^{th}	8^{th}
-4	0	0	0	1/280
-3	0	0	-1/60	-4/105
-2	0	1/12	3/20	1/5
-1	-1/2	-2/3	-3/4/	-4/5
0	0	0	0	0
1	1/2	2/3	3/4/	4/5
2	0	-1/12	-3/20	-1/5
3	0	0	1/60	4/105
4	0	0	0	-1/280

Error

- A field variable ϕ can be represented as a Fourier Series. A given term (mode) n of the series is

$$\phi_n(x) = c_n \cdot e^{i\kappa_n x} \qquad \textbf{(1)}$$

  - κ_n is the wavenumber of the mode
  - c_n is the complex coefficient (amplitude)
- The exact derivative is $\dfrac{d\phi}{dx} = c_n i\kappa e^{i\kappa x}$ **(2)**
- The discrete approx. is $\left(\dfrac{\delta\phi}{\delta x}\right)_j = \dfrac{\sum_k a_k \phi_{j+k}}{\Delta x}$ **(3)**

- Sub (1) into (3) and compare to (2). The rel. error is

$$\epsilon = 1 - \frac{2}{\kappa\Delta x}\sum_{k\geq 0} a_k \sin(\kappa\Delta x \cdot k)$$

Error

- The number of grid intervals to span half a wave is

$$N_{1/2} = \left\lceil \frac{\pi}{\kappa \Delta x} \right\rceil$$

- For 0.1% error, 2nd, 4th, 6th, 8th order derivatives we need 41.9, 7.7, 4.3, 3.2 grid intervals across a half-wave: